Lorentz invariance of scalar field action on κ-Minkowski space-time

Sebastian Nowak*

Institute for Theoretical Physics
University of Wrocław
Pl. Maxa Borna 9
Pl–50-204 Wrocław, Poland

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Abstract

We construct field theory on noncommutative κ-Minkowski space-time. Having the Lorentz action on the noncommutative space-time coordinates we show that the field lagrangian is invariant. We show that noncommutativity requires replacing the Leibnitz rule with the coproduct one.

1 Introduction

κ-Minkowski space-time with no-trivial commutator between time and space being

$$[x_0, x_i] = -\frac{i}{\kappa}\, x_i \tag{1}$$

has been first constructed in [1], [2] as a Hopf algebra dual to the translational part of κ-Poincaré algebra [3]. After Doubly Special Relativity (DSR) has been formulated in [4], [5], [6], it was soon realized [7] that κ-Minkowski space-time is a natural candidate for space-time of DSR[1]. This space-time arises also naturally in investigations of 2+1 (quantum) gravity [10], [11], giving rise to the claim that it is the space-time emerging from semiclassical, weak field approximation of 3+1 gravity as well.

It seems natural to investigate properties of classical and quantum fields on κ-Minkowski space-time. Indeed such investigations has been undertaken, among others in [12], [13], [14], [15], [17], [16]. It has been shown [19] that the invariance of action on scalar fields on κ-Minkowski leads to deformed algebra of

*e-mail: `pantera@ift.uni.wroc.pl`

[1]For up to date reviews of DSR program see [8], [9].

symmetries and some nontrivial co-algebra structure. In this paper we proceed in different manner, we consider Lorentz action on generators of κ-Minkowski space and compute Lorentz action on normally ordered plane wave and then we show Lorentz invariance of κ-deformed Klein-Gordon action.

2 κ-deformed algebra of symmetries

In this section we review the construction of κ-Poincaré algebra [3] which can be seen as deformed algebra of symmetries. Lorentz sector of this algebra is standard but the action of boost generators on momenta is deformed. Here we write down this algebra in so-called bicrossproduct basis [2]

$$[M_i, M_j] = i\,\epsilon_{ijk}M_k, \quad [M_i, N_j] = i\,\epsilon_{ijk}N_k,$$
$$[N_i, N_j] = -i\,\epsilon_{ijk}M_k. \tag{2}$$
$$[M_i, p_j] = i\,\epsilon_{ijk}p_k, \quad [M_i, p_0] = 0$$
$$[N_i, p_j] = i\,\delta_{ij}\left(\frac{\kappa}{2}\left(1 - e^{-2p_0/\kappa}\right) + \frac{\mathbf{p}^2}{2\kappa}\right) - \frac{i}{\kappa}p_i p_j, \;\; [N_i, p_0] = i\,p_i. \tag{3}$$

This algebra together with coalgebra structure written below forms noncommutative, noncocommutative Hopf algebra

$$\triangle(M_i) = M_i \otimes 1 + 1 \otimes M_i$$
$$\triangle(N_i) = N_i \otimes 1 + e^{-p_0/\kappa} \otimes N_i + \frac{1}{\kappa}\epsilon_{ijk}p_j \otimes M_k$$
$$S(N_i) = -e^{p_0/\kappa}(N_i - \frac{1}{\kappa}\epsilon_{ijk}p_j M_k), \;\; S(M_i) = -M_i \tag{4}$$
$$\triangle(p_i) = p_i \otimes 1 + e^{-p_0/\kappa} \otimes p_i$$
$$\triangle(p_0) = p_0 \otimes 1 + 1 \otimes p_0$$
$$S(p_i) = -p_i e^{p_0/\kappa}, \;\; S(p_0) = -p_0 \tag{5}$$

Following the general scheme we can introduce κ-Minkowski space T^* as a dual Hopf algebra to algebra of translations (momenta). With the use of pairing

$$< p_\mu, x_\nu >= -i\eta_{\mu\nu} \tag{6}$$

together with axioms of a Hopf algebra duality

$$< t, xy >=< t_{(1)}, x >< t_{(2)}, y >,$$
$$< ts, x >=< t, x_{(1)} >< s, x_{(2)} >,$$
$$\forall t, s \in T, \quad x, y \in T^* \tag{7}$$

we get

$$[x_i, x_j] = 0, \quad [x_0, x_i] = -\frac{i}{\kappa}x_i$$

$$\triangle x_\mu = x_\mu \otimes 1 + 1 \otimes x_\mu . \tag{8}$$

where we use Sweedler notation for coproduct

$$\triangle t = \sum t_{(1)} \otimes t_{(2)} .$$

Having the Hopf algebra of space-time coordinates we can write down the covariant action of T on it:

$$t \triangleright x =< x_{(1)}, t > x_{(2)}, \quad \forall x \in T^*, \quad t \in T. \tag{9}$$

In our case it reads

$$p_i \triangleright : \psi(x_i, x_0) := \frac{1}{i} : \frac{\partial}{\partial x_i} \psi(x_i, x_0) :,$$

$$p_0 \triangleright : \psi(x_i, x_0) := -\frac{1}{i} : \frac{\partial}{\partial x_0} \psi(x_i, x_0) : \tag{10}$$

which is like in classical case but remembering normal ordering.

The action of U(so(1,3)) generators on momenta, defined in formula (3) (we denote it here for simplicity by $\triangleleft$) , can be translated by duality into generators of κ-Minkowski space. We have

$$< t, h \triangleright x >=< t \triangleleft h, x >$$

$$\forall t \in T, \quad h \in U(so(1,3)), \quad x \in T^* . \tag{11}$$

Using duality pairing written in (6) we get

$$M_i \triangleright x_0 = 0, \quad M_i \triangleright x_j = i\epsilon_{ijk} x_k ,$$

$$N_i \triangleright x_0 = i x_i, \quad N_i \triangleright x_j = i\delta_{ij} x_0 . \tag{12}$$

We see that the action is the same as in classical case. The difference occurs while acting on product of coordinates. In noncommutative case we must use the coproduct formula instead of usual Leibnitz rule so as no contradiction with relation

$$[x_0, x_i] = -\frac{i}{\kappa} x_i$$

arises. We have

$$h \triangleright (xy) = (h_{(1)} \triangleright x)(h_{(2)} \triangleright y) \tag{13}$$

Moreover from the structure of bicrossproduct construction it appears that one can also define the action of space-time algebra on Lorentz algebra and then define the cross relations between them [2]. In our case it has following form

$$[M_i, x_0] = 0, \quad [M_i, x_j] = i\epsilon_{ijk} x_k ,$$

$$[N_i, x_0] = i x_i - \frac{i}{\kappa} N_i, \quad [N_i, x_j] = i\delta_{ij} x_0 - \frac{i}{\kappa} \epsilon_{ijk} M_k . \tag{14}$$

It has been shown that so(1,3) algebra and space-time algebra (7) together with above cross relations form so(1,4) algebra. Moreover the action on momenta written in equation (3) may be derived from the action of so(1,4) algebra on four dimensional de Sitter space of momenta [23].

3 Integration and Fourier transform

In order to define Fourier transform on κ-Minkowski we have to define normally ordered plane wave, which is a solution of κ-deformed Klein-Gordon field equation [17]. Normal ordering is necessary because space-time coordinates do not commute, here we choose "time to the right" ordering

$$: e^{ipx} := e^{i\mathbf{px}} e^{-ip_0 x_0} \tag{15}$$

which simply means all x_0 shifted to the right. Having normally ordered plane wave one can define κ-deformed Fourier transform describing fields on noncommutative space:

$$\Phi(x_0, \mathbf{x}) = \frac{1}{(2\pi)^4} \int d\mu \Phi(p_0, \mathbf{p})\, e^{i\mathbf{px}}\, e^{-ip_0 x_0}, \tag{16}$$

where

$$d\mu = d^4 p\, e^{3p_0/\kappa} \tag{17}$$

is the measure invariant under the action of U(so(1,3)) algebra. The field written above is an element of Hopf algebra described in equations (8). One can equip this algebra with hermitian conjugation. This implies the conjugation of field:

$$\Phi^+(x_0, \mathbf{x}) = \frac{1}{(2\pi)^4} \int d\mu \Phi^\star(p_0, \mathbf{p})\, e^{ip_0 x_0}\, e^{-i\mathbf{px}}, \tag{18}$$

which can be rewritten in the following form:

$$\Phi^+(x_0, \mathbf{x}) = \frac{1}{(2\pi)^4} \int d\mu \Phi^\star(p_0, \mathbf{p})\, e^{iS(\mathbf{p})\mathbf{x}}\, e^{-iS(p_0)x_0}, \tag{19}$$

where we used equality

$$e^{ip_0\, x_0} e^{-i\mathbf{p}\,\mathbf{x}} = e^{-i\mathbf{p}e^{p_0/\kappa}\,\mathbf{x}} e^{ip_0\, x_0} \tag{20}$$

together with definition of antipode S (equation (5)).

In order to define the field theory on κ-Minkowski we need to define κ-deformed integration. To end this we must introduce, in analogy to the classical case, the following formula

$$\frac{1}{(2\pi)^4} \int\int d^4x : e^{ipx} := e^{3p_0/\kappa} \delta^4(p) \tag{21}$$

This formula is invariant under the action of Lorentz generators on momentum space defined in equations (3) . Let us check it for boost transformation; we have

$$\frac{1}{(2\pi)^4} \int\int d^4x : e^{ip'x} := e^{3p_0'/\kappa} \delta^4(p') = e^{3p_0'/\kappa} J(\frac{\partial p_\mu'}{\partial p_\nu}) \delta^4(p) = e^{3p_0/\kappa} \delta^4(p) \tag{22}$$

and the same result holds for rotation generators.

Using the above definition of delta function one can find

$$\int d^4x \Psi(x)\Phi(x) =$$

$$\int d^4x \int d\mu d\mu' \ \tilde{\Psi}(p)\tilde{\Phi}(p') \ : e^{ipx} :: e^{ip'x} :=$$

$$\int d\mu \, d\mu' \, \tilde{\Psi}(p) \, \tilde{\Phi}(p') \, \delta(p \dot{+} p') \tag{23}$$

To derive this formula one uses the fact that

$$: e^{ipx} :: e^{ip'x} :=: e^{i(p \dot{+} p')x} : \tag{24}$$

where

$$p \dot{+} p' = (p_0 + p_0'; \mathbf{p} + e^{-p_0/\kappa} \, \mathbf{p}') \tag{25}$$

represents the "co-product summation rule", related to the Hopf algebra structure of κ-Poincare algebra.

We can define the scalar product of fields on κ-Minkowski as

$$(\Phi(x), \Psi(x)) = \int d^4x \, \Phi^{+}(x) \, \Psi(x) = \int d^4p \, e^{3p_0} \, \Phi^{*}(p)\Psi(p) \tag{26}$$

Now it is a good point to introduce Lorentz action on fields on noncommutative space-time. We start with normally ordered plane wave. Using the Lorentz action on noncommuting coordinates (12) together with coproduct rule (13) we find

$$M_i \triangleright (e^{ip_jx_j} e^{-ip_0x_0}) = \epsilon_{ijk} x_j p_k e^{ip_jx_j} e^{-ip_0x_0}$$

$$N_i \triangleright (e^{ip_jx_j} e^{-ip_0x_0}) =$$

$$e^{ip_jx_j} \left[\left(\delta_{ij} (\frac{\kappa}{2}(1 - e^{-\frac{2P_0}{\kappa}}) + \frac{1}{2\kappa}\vec{p}^2) - \frac{1}{\kappa} p_i p_j \right) x_j + (-p_i x_0) \right] e^{-ip_0x_0} \tag{27}$$

and together with action of translation generators defined in equation (10) we have the action of the κ-Poincaré algebra on normal ordered plane wave. One can easily show that the generators of this action are hermitian with respect to scalar product (26). One can also find differential realization of the above action [17], which turns out to be nonlinear.

Now taking infinitesimal parameter ε we can introduce infinitesimal Lorentz transformations on normal ordered plane waves.

$$(1 + i\varepsilon M_i) \triangleright (e^{ip_jx_j} e^{-ip_0x_0}) = e^{ip_j'x_j} e^{-ip_0x_0}$$

where

$$p_j' = p_j - i\varepsilon[M_i, p_j] \tag{28}$$

and

$$(1 + i\varepsilon N_i) \triangleright (e^{ip_jx_j} e^{-ip_0x_0}) = e^{ip_j'x_j} e^{-ip_0'x_0}$$

$$p'_j = p_j - i\varepsilon[N_i, p_j], \quad p'_0 = p_0 - i\varepsilon[N_i, p_0] \tag{29}$$

We see that our normally ordered plane wave is an analog of classical plane wave e^{ipx}. The action on space-time coordinates interchanges with the action on energy-momentum space. Moreover in both cases one can see the phase space as the Lee algebra of space-time coordinates and the energy-momentum Hopf algebra as the algebra of functions on group generated by the algebra of space-time coordinates. The significant difference is that the classical algebra is commutative while the quantum is not.

Knowing the action of Lorentz algebra on plane wave we can write down the transformations of Fourier transform (16)

$$(1 + i\varepsilon N_i) \triangleright \Phi(x, t) = \int d\mu \Phi(p_0, \vec{p}) e^{ip'_j x_j} e^{-ip'_0 x_0}. \tag{30}$$

Now we can change variables under the integral and since the measure $d\mu$ is invariant under the action of U(so(1,3)) algebra we get

$$(1 + i\varepsilon N_i) \triangleright \Phi(x, t) = \int d\mu \Phi(p'_0, \vec{p'}) e^{ip_j x_j} e^{-ip_0 x_0} \tag{31}$$

where

$$p'_j = p_j + i\varepsilon[N_i, p_j], \quad p'_0 = p_0 + i\varepsilon[N_i, p_0] \tag{32}$$

but on the other side we can write

$$(1 + i\varepsilon N_i) \triangleright \Phi(x, x_0) = \Phi'(x, x_0) = \int d\mu \Phi'(p_0, \vec{p}) e^{ip_j x_j} e^{-ip_0 x_0}. \tag{33}$$

Compering equations (31) and (33) we see that

$$\Phi'(p'_0, \vec{p'}) = \Phi(p_0, \vec{p})$$

which means that fields in energy-momentum space are scalar fields. Here we consider only the action of algebra but one can also consider the action of κ-Poincaré group on fields. This was done in paper [14], with the help of κ-Wigner construction.

Using the first order Taylor expansion we can write equation (31) in the following form

$$(1 + i\varepsilon N_i) \triangleright \Phi(x, t) = \int d\mu (1 + i\varepsilon N_i) \Phi(p_0, \vec{p}) e^{ip_j x_j} e^{-ip_0 x_0}, \tag{34}$$

where the boost generator on the right hand side stands for differential operator in momentum space. Having the above transformation rules we see that variables p_0, p_i may be indeed identified with energy and momentum in bicrossproduct basis.

Another important thing to notice is the action of κ-Poincaré algebra on product of fields. To end this we must apply the coalgebra structure, which in

classical case, where the coproducts are trivial, is just the Leibnitz rule. The action of Lorentz algebra is due to the covariance condition (13). We get

$$N_i \triangleright (\Phi(x)\Psi(x)) = \int\int d^4p\, e^{3p_0/\kappa}\, d^4k\, e^{3k_0/\kappa}\, \Big[(N_i\Phi(p))\Psi(k) +$$

$$e^{-p_0/\kappa}\Phi(p)(N_i\Psi(k)) + \epsilon_{ijk}p_j\Phi(p)(M_k\Psi(k))\Big] : e^{ipx} :: e^{ikx} : . \tag{35}$$

From the above equation it is seen that noncommutativity of space-time enforces applying the coproduct rule on fields in momentum space [18]. The action of translation generators is due to the equation (10) and it reads

$$p_0 \triangleright (\Phi(x)\Psi(x)) = \int\int d^4p\, e^{3p_0/\kappa}\, d^4k\, e^{3k_0/\kappa}\, (p_0 + k_0)\Phi(p)\Psi(k) : e^{ipx} :: e^{ikx} :$$

$$p_i \triangleright (\Phi(x)\Psi(x)) = \int\int d^4p\, e^{3p_0/\kappa}\, d^4k\, e^{3k_0/\kappa}\, (p_i + e^{-p_0/\kappa}k_i)\Phi(p)\Psi(k) : e^{ipx} :: e^{ikx} : \tag{36}$$

which is nothing but the coproduct summation rule. In the next section we'll see that we can also define invariant action and field equation.

4 Actions and field equations

We consider here only the free κ-deformed KG (Klein-Gordon) action which has the following form [13]:

$$S = \frac{1}{2}\int d^4x\, \Big(\eta^{\mu\nu}\, \Phi(x)\, \hat{\partial}_\mu \hat{\partial}_\nu\, \Phi(x) - \mathcal{M}^2\, \Phi(x)^2\Big) . \tag{37}$$

where $\hat{\partial}_\nu$ means covariant differentiation on noncommuting space-time [20], [21], [22]. We consider only the real fields ($\Phi^+(x) = \Phi(x)$). By construction the action should be invariant under action of deformed κ-Poincaré algebra. Let us show this invariance explicitly. To do that we write the action in somewhat convenient form

$$S = \int d^4x \int\int d^4p\, e^{3p_0/\kappa}\, d^4k\, \Phi(p)\Phi(S(k)) : e^{ipx} :: e^{iS(k)x} : \mathcal{M}(k) \tag{38}$$

where $\mathcal{M}(k)$ is the mass shell condition and will be explicitly written below. Using formula (35) we have

$$(1 + i\varepsilon N_i) \triangleright S = \int d^4x \int\int d^4p\, e^{3p_0/\kappa}\, d^4k$$

$$\Big[\Phi(p)\Phi(S(k)) + (i\varepsilon N_i\Phi(p))\Phi(S(k)) + e^{-p_0/\kappa}\Phi(p)\,(-i\varepsilon S(N_i)\Phi(S(k))\,)$$

$$+\epsilon_{ijk}p_j\Phi(p)\,(-i\varepsilon S(M_k)\Phi(S(k))\,)\,]\, \mathcal{M}(k) : e^{ipx} :: e^{iS(k)x} : . \tag{39}$$

Now using the following formulas for antipodes S

$$S(M_i) = -M_i, \;\; S(N_i) = -e^{p_0/\kappa}(N_i - \frac{1}{\kappa}\epsilon_{ijk}p_j M_k) \tag{40}$$

together with definition of delta function (21) we get

$$(1 + i\varepsilon N_i) \triangleright S = \int d^4p \, e^{3p_0/\kappa} \, (1 + \varepsilon N_i) \, (\Phi(p)\Phi(S(p)) \,) \, \mathcal{M}(k) = S \tag{41}$$

where we made use of the invariance of the measure. We get the same result if we repeat the above calculations for generators of rotations.

Let us now turn to the remaining part of the (deformed) Poincaré symmetry, namely the symmetries with respect to the space time translations. We have

$$\varepsilon P_\mu \triangleright S = \int d^4x \int \int d^4p \, e^{3p_0/\kappa} \, d^4k \, e^{3k_0/\kappa}$$

$$\varepsilon(p_\mu \dot{+} k_\mu)\Phi(p)\Psi(k)\mathcal{M}(k) : e^{ipx} :: e^{ikx} := 0 \tag{42}$$

where we used definition of delta function. following the same procedure we can show invariance of the scalar product (26).

In terms of the Fourier transformed fields the action reads

$$S = \frac{1}{2} \int \, dp_0 d^3\mathbf{p} \, \Phi(p_0, \mathbf{p}) \, \Phi(-p_0, -e^{p_0/\kappa} \, \mathbf{p})$$

$$\left[\kappa^2 \sinh^2 \frac{p_0}{\kappa} - \frac{1}{2} \, \mathbf{p}^2 \left(e^{2p_0/\kappa} + 1\right) + \frac{\mathbf{p}^4}{4\kappa^2} \, e^{2p_0/\kappa} - M^2\right] \tag{43}$$

where we used the explicit form of $\mathcal{M}(p_0, \mathbf{p})$

$$\mathcal{M}(p_0, \mathbf{p}) \equiv \left[\kappa^2 \sinh^2 \frac{p_0}{\kappa} - \frac{1}{2} \, \mathbf{p}^2 \left(e^{2p_0/\kappa} + 1\right) + \frac{\mathbf{p}^4}{4\kappa^2} \, e^{2p_0/\kappa} - M^2\right] \tag{44}$$

Note that the factors $e^{3p_0/\kappa}$ in the integration measure cancel. This action can be also expressed in slightly more compact way using the antipode S (generalized "minus") of the κ-Poincaré algebra defined in equations (5)

$$S = \frac{1}{2} \int \, dp_0 d^3\mathbf{p} \, \Phi(p_0, \mathbf{p}) \, \Phi(S(p_0), S(\mathbf{p})) \, \mathcal{M}(p_0, \mathbf{p}) \tag{45}$$

Varying this action with respect to $\Phi(p_0, \mathbf{p})$ and noting that

$$\mathcal{M}(p_0, \mathbf{p}) = \mathcal{M}(S(p_0), S(\mathbf{p}))$$

we find the on shell condition of the form

$$\kappa^2 \sinh^2 \frac{p_0}{\kappa} - \frac{1}{2} \, \mathbf{p}^2 \left(e^{2p_0/\kappa} + 1\right) + \frac{\mathbf{p}^4}{4\kappa^2} \, e^{2p_0/\kappa} - M^2 = 0 \tag{46}$$

Now we can write down the real field on shell in the following form

$$\Phi(x) = \int d^3k\, a_k^* e^{i\mathbf{kx}} e^{-ik_0x_0} + \int d^3k\, a_k e^{iS(\mathbf{k})\mathbf{x}} e^{-iS(k_0)x_0} \tag{47}$$

which is very similar to the classical case. The difference is the antipode S in exponent instead of minus. On the mass shell we have

$$e^{k_0/\kappa} = \frac{1}{1 - \frac{|\mathbf{k}|}{\kappa}}. \tag{48}$$

where for simplicity we consider only the massless case.

5 conclusions

We have shown that in analogy to the classical case, for scalar fields on κ-Minkowski one can construct scalar product and Hermitian action of κ-Poinaré algebra generators. A subalgebra of this algebra can be interpreted as infinitesimal Lorentz transformations, under which the field action is invariant. The solutions of κ-deformed Klein-Gordon equation have the form similar to classical field.

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